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Heterogenous intertemporal elasticity of substitution and relative risk aversion: estimation of optimal consumption choice with habit formation and measurement errors

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Abstract

This paper investigates the existence and degree of variation across households and over time in the intertemporal elasticity of substitution (IES) and the coefficient of relative risk aversion (RRA) that is generated by habit forming preferences. To do so, we develop a new nonlinear GMM estimator to investigate the presence of habit formation in household consumption using data from the Panel Study of Income Dynamics. Our method accounts for classical measurement errors in consumption without parametric assumptions on the distribution of measurement errors. The estimation results support habit formation in food consumption. Using these estimates, we develop bounds for the expectation of the implied heterogenous intertemporal elasticity of substitution and relative risk aversion that account for measurement errors and compute asymptotically valid confidence intervals on these bounds. We find that these parameters display significant variation across households and over time.
KEYWORDS: Intertemporal elasticity of substitution; Relative risk aversion; Habit formation; Classical measurement errors; Nonlinear models.
JEL C13, C33, D12, D91, E21

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Highlights

- We test whether the IES and the RRA are not constant across individuals and over time.
- We use habit formation preferences that generate variation in the IES and the RRA.
- We develop an estimator that accounts for classical measurement errors in the data.
- The estimation results support habit formation in household food consumption.
- We find that the IES and the RRA display variation across households and over time.

1 Introduction

Until recently, little was known about differences in intertemporal substitution and risk aversion across groups of individuals and over time. Recent developments in the analysis of life cycle consumption and savings emphasize the presence of heterogeneity in these parameters and their implications for economic policy.[1] The dominant empirical approach to investigating heterogeneity in these parameters, using microdata, is to start with models that imply constant intertemporal elasticity of substitution (IES) or relative risk aversion (RRA). These models are either analyzed for different economic units, or heterogeneity in these parameters is explicitly taken into account during estimation. The former approach was undertaken by Attanasio and Weber (1993), Vissing-Jorgenssen (2002) and Crossley and Low (2011) in order to analyze heterogeneity in the IES. The latter approach was undertaken by Alan and Browning (2010) to analyze heterogeneity in the RRA. Other approaches include survey-based analysis (Barsky et al., 1997; Guiso and Paiella, 2006; and Eisenhauer and Ventura, 2003) and experiment-based elicitation (Andersen et al., 2010).

An alternative approach for investigating heterogeneity in the IES and RRA is to directly model preferences that generate such heterogeneity. In particular, some habit forming preferences deliver individual and time varying IES and RRA. In this paper, we exploit this property of habit formation models in order to derive inferences about the IES and RRA. To do so, we empirically investigate the presence of internal habit formation in household food consumption using data from the Panel Study on Income Dynamics (PSID). The advantage of this approach is that heterogeneity in the IES and RRA are determined by preference parameters that are not functions of economic environment, making these models more suitable for counterfactual policy analysis. For example, if groups of individuals are formed as a function of the economic environment, then models that estimate group-

[1] See Alan and Browning (2010) and the references therein for discussions on the recent developments in investigating individual heterogeneity in the relative risk aversion.

specific constant IES parameters are not useful for counterfactual policy analysis that changes the economic environment.

In the representative agent framework, the empirical evidence largely supports the existence of habit formation in consumption.[2] However, in micro data the evidence of habit formation is inconclusive. The studies of Carrasco et al. (2005) and Browning and Collado (2007) support habit formation in food consumption, while those of Meghir and Weber (1996) and Dynan (2000) do not. Comparisons between these results are confounded by differences in preference specification, the approximations employed to obtain estimating equations, and differences in the data. Carrasco et al. (2005) apply the test proposed by Meghir and Weber on a different data set of the same frequency, but longer periods. They argue that this is the main reason for their contrasting conclusions. In this paper, we employ the same data set as in Dynan, that is, household food consumption data from the PSID. However, we find evidence of significant habit formation in food consumption. We argue that the difference between ours and Dynan's result stems mainly from the differences in the estimation approach as well as the treatment of measurement errors in consumption. The use of the PSID along with our model specification make our results directly comparable not only with micro studies such as Dynan, but also with the large body of macroeconomic literature.

We assume that habit formation takes a multiplicative (or ratio) form.[3] The choice of the multiplicative specification is motivated by two related points. First, individual consumption data is more volatile than aggregate consumption data. As a result, while the restriction of positive consumption services is relatively easy to satisfy in the difference specification when using aggregated data, it is likely to be violated in micro data when the difference specification is assumed. The fact

[2]See Fuhrer (2000), Chen and Ludvigson (2009), and Smith and Zhang (2007) for later examples.

[3]In our analysis, we assume that individual preferences over food consumption are characterized by the habit formation specification introduced in Abel (1990), where consumption services is given by $\tilde{c}_t = c_t/c_{t-1}^{\alpha}$. The main alternative to this specification is the difference model of habit. In this case, consumption services is given by, for example $\tilde{c}_t = c_t - \alpha c_{t-1}$.

that the multiplicative specification of consumption services satisfies this positivity constraint for any pair (c_t, c_{t-1}) makes it more appropriate when using micro data. Second, under the multiplicative specification, the consumption Euler equation belongs to a class of moment conditions for which we develop a method for controlling for classical measurement errors without imposing parametric assumptions about their distribution.[4] We then exploit the structure of the Euler equation to develop a nonlinear generalized method of moments (GMM) estimator. Our estimator extends Alan et al. (2009), who propose two exact nonlinear GMM estimators for the consumption Euler equation without habit formation. To the best of our knowledge, this paper presents the first exact Euler equation nonlinear GMM method that is developed to investigate the existence of habit formation without imposing parametric assumptions on the distribution of measurement errors. We provide sufficient conditions for identification of the structural parameters of interest: the time-discount factor, the utility curvature parameter, and the habit formation parameter. A Monte Carlo simulation exercise shows that the proposed estimator performs well in recovering the parameters of interest. The simulation exercise also shows that not accounting for measurement errors leads to overestimation of the utility curvature parameter and underestimation of the strength of habits and the time-discount factor.

Under the assumption that the observed consumption is measured with error, the implied household- and time-specific IES and RRA cannot be directly inferred. Furthermore, the expectations of the IES and the RRA are not point identified. We develop bounds on the IES and RRA that account for measurement errors and compute asymptotically valid confidence intervals for these bounds using the parameter estimates from the model. The bounds for the IES support typical findings in the literature. The 95% confidence interval for the IES is [0.083, 0.193]. The corresponding 95% confidence interval for the RRA is [4.991, 13.226]. This is somewhat higher than the prevailing estimates in empirical studies of consumption models without habit formation.

[4] For discussion of the measurement error issue in consumption data see Shapiro (1984) and Runkle (1991).

However, the growing literature on heterogeneity in the RRA provides evidence that this parameter can be between 2 and 16 if one accounts for individual heterogeneity in risk aversion (examples are Barsky et al., 1997; Eisenhauer and Ventura, 2003; Guiso and Paiella, 2006; and Alan and Browning, 2010).

Post-estimation analysis shows that these parameters display significant variation across individuals and over time. The IES is decreasing and convex in age; the RRA is increasing and concave in age; the IES depicts a U shape in income; and the RRA depicts a dome shape in income.

The remainder of the paper is as follows. Section 2 describes the model. Section 3 presents the estimator and discusses identification of the parameters of interest. Section 4 investigates the small sample properties of the proposed estimator. Section 5 describes the data sample. Section 6 present the empirical results. Section 7 examines the implications for the relative risk aversion and intertemporal elasticity of substitution, and Section 8 concludes. The proofs and detailed derivations are presented in the Appendix.

2 Theoretical Framework

Household i chooses a sequence of consumption $\{c_{is}, s = t, \cdots, T\}$ to maximize its expected lifetime utility function, given by

$$E_{it}\sum_{s=t}^{T}\beta^{s-t}\phi_{is}\frac{\tilde{c}_{is}^{1-\gamma}-1}{1-\gamma}, \tag{2.1}$$

where the expectation is conditional on all relevant information for household i at time t, $\beta \in (0,1)$ is the time-discount factor, γ the utility curvature parameter, and $\tilde{c}_{it}$ denotes consumption services in period t. Consumption services is defined as the ratio between current consumption expenditures and past consumption expenditures, geometrically weighted:

$$\tilde{c}_{it} = \frac{c_{it}}{c_{it-1}^{\alpha}}, \tag{2.2}$$

where $0 \leq \alpha \leq 1$ measures the strength of habits: $\alpha = 1$ denotes the strongest and $\alpha = 0$ indicates no habit in consumption.

The importance of augmenting the individual utility function with individual-specific taste shifters has been widely accepted in the estimation of optimal consumption choices using micro data. Household-specific "taste shifters" ϕ_{it} are given by

$$\phi_{it} = \exp(\delta w_{it} + \omega_i), \tag{2.3}$$

where w_{it} is a vector of exogenous time-varying observed household characteristics and ω_i is a household fixed effect.

We assume that household i is not subject to liquidity constraints and has rational expectations. The first-order necessary condition for the household's optimization problem is

$$E\left[\beta(1 + r_{it+1})MU_{it+1} - MU_{it}|z_{it}\right] = 0, \tag{2.4}$$

where r_{it+1} is the rate of return available to household i between periods t and $t+1$, z_{it} denotes the set of all information that is available to household i at time t, and MU_{it} represents household i's marginal utility of consumption in period t:

$$MU_{it} = \frac{\phi_{it}}{c_{it}}\left(\frac{c_{it}}{c_{it-1}^{\alpha}}\right)^{1-\gamma} - \alpha\beta\frac{\phi_{it+1}}{c_{it}}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}. \tag{2.5}$$

Notice that if $\alpha = 0$, MU_{it} in equation (2.5) reduces to the marginal utility of time separable models. For $\alpha > 0$, consumption services are negatively related to past consumption levels. This property is shared with difference models of habit formation with positive α (i.e., $\tilde{c}_{it} = c_{it} - \alpha c_{it-1}$). However, in the case of the multiplicative model, $\alpha > 0$ is not sufficient to characterize habit formation. The multiplicative model also requires $\gamma > 1$ in order to exhibit habit formation. Indeed, as long as both $\alpha > 0$ and $\gamma > 1$, the household's marginal utility of consumption in period t is an increasing function of period $t-1$ consumption, yielding a complementarity effect of consumption over time.

Substituting equation (2.5) into equation (2.4) obtains the following moment condition:

$$E\left[\beta(1+r_{it+1})\left(\frac{\phi_{it+1}}{c_{it+1}}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}-\alpha\beta\frac{\phi_{it+2}}{c_{it+1}}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha}}\right)^{1-\gamma}\right)-\frac{\phi_{it}}{c_{it}}\left(\frac{c_{it}}{c_{it-1}^{\alpha}}\right)^{1-\gamma}+\alpha\beta\frac{\phi_{it+1}}{c_{it}}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}\Bigg|z_{it}\right]=0 \tag{2.6}$$

3 The Estimator

In order to derive an exact nonlinear GMM estimator of the parameter vector of interest from equation (2.6), we address two key issues: potential household-specific effects in consumption and measurement errors in consumption. We discuss these issues in turn.

3.1 Consumption growth

Habit formation in consumption generates positive serial correlation in consumption over time, as does household-specific heterogeneity in consumption. As a result, not accounting for household-specific heterogeneity in consumption series will bias the estimates in favor of finding evidence of habit formation. Therefore, to eliminate potential household-specific effects, we transform the moment equation (2.6) into one that is expressed in terms of the growth rate of consumption. Let

$g_{it} = c_{it}/c_{it-1}$, and $\varphi_{it} = \phi_{it}/\phi_{it-1}$. Because c_{it}, c_{it-1}, w_{it}, and ω_i are known to household i in period t, so is

$$\frac{\phi_{it}}{c_{it}}\left(\frac{c_{it}}{c_{it-1}^{\alpha}}\right)^{1-\gamma},$$

which is strictly positive for all values of c_{it}, c_{it-1}, w_{it}, and ω_i. Thus, dividing equation (2.6) by this quantity leads to

$$E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\left(1-\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}\right)-\left(1-\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\right)\middle|z_{it}\right]=0. \tag{3.1}$$

The growth rate transformation of the Euler equation also has the advantage of eliminating unobserved household fixed effects in the taste shifters, ω_i, so that $\varphi_{it} = \exp(\delta\Delta w_{it})$. Similar to the first difference transformation in linear panel data models, the growth rate transformation comes at the cost of possibly reducing the precision of the estimates. However, for our purpose, the potential benefits of the growth rate transformation in terms of robustness outweigh the potential cost in terms of loss of precision. Additionally, the growth rate transformation also has the advantage of eliminating any household-specific unobserved effects in consumption and in income measurement errors. We return to this point in the next section.

3.2 Measurement error

Given a set of appropriate instruments and the absence of measurement errors, consistent estimators of the parameters α, β, γ, and δ can be obtained based on the moment condition in equation (3.1). However, the estimation of nonlinear rational expectation models using micro data is complicated by the problem of measurement errors in consumption, which, if ignored, will likely result in inconsistent estimation of the key parameters of interest.

Log-linearization of the Euler equation has the advantage of remaining tractable when accounting for measurement errors in consumption in both time-separable and nonseparable models.[5] However, as discussed in Carroll (2001), a log-linear approximation can result in severe bias of the parameter estimates.[6] Nonlinear GMM estimators based on the Euler equation provide an alternative to log-linearization. Still, the problem of measurement errors remains difficult without additional distributional assumptions. Significant progress has been made in accounting for classical measurement error in time-separable models. Ventura (1994) assumes that measurement errors are serially independent and lognormally distributed, while Hong and Tamer (2003) assume that the measurement errors are independent and that their marginal distributions are Laplace with zero mean and unknown variance. After re-parametrization, the approaches in Ventura (1994) and Hong and Tamer (2003) (applied to the time-separable Euler equation) yield similar moment conditions for the estimation of the utility curvature parameter, subject to a proper set of instruments. However, the time-discount factor remains unidentified. Alan et al.(2009) suggest two exact GMM estimators: one assumes a lognormal distribution for the measurement errors, and the other relaxes this assumption. The advantage of the lognormal assumption is that it allows for identification of the measurement error variance along with other structural parameters of the model.

Nonseparabilities in preferences add another layer of difficulty to nonlinear estimation with classical measurement errors. Due to the increasing complexity of the moment conditions in the presence of habit formation, measurement errors cannot be easily separated from observed consumption. Thus, to our knowledge, there are no studies that use nonlinear estimators to test for time-nonseparabilities in individual preferences over consumption, accounting for measurement

[5]Log-linearization of the Euler equation allows Dynan (2000) to account for measurement errors in consumption expenditures without additional parameterization while testing for nonseparabilities in current and past household consumption.

[6]Attanasio and Low (2004) show that the estimation of a log-linearized consumption model can yield consistent estimates of the parameters when data covers very long time period.

errors.[7]

Let true consumption c_{it} be measured with a multiplicative error $\tilde{\eta}_{it}$, so that observed consumption is given by $c^o_{it} = c_{it}\tilde{\eta}_{it}$, where $\tilde{\eta}_{it} > 0$. It is interesting to note that the transformation of the Euler equation into one that is expressed in terms of growth rates also eliminates household-specific, time-invariant measurement errors in consumption. The assumptions on the measurement errors are therefore presented conditional on these household-specific effects. Suppose that the measurement errors can be decomposed as $\tilde{\eta}_{it} = \mu_i \eta_{it}$.

Assumption 3.1. *Given μ_i and for each t, η_{it} is stationary and independent from the time vector of consumption, the taste shifters, the information set, the interest rate, and income.*

Assumption 3.1 is an extension of the classical errors-in-variables independence assumption. Note that the individual-specific effect, μ_i, is allowed to be arbitrarily correlated with the time vector of consumption, the taste shifters, the information set, the interest rate, and income. It is also allowed to be correlated with η_{it} for all t. Ignoring μ_i, Assumption 3.1 is similar to that made in Hong and Tamer (2003). However, unlike in Hong and Tamer (2003), the method developed here does not impose parametric restrictions on the distribution of measurement errors. Furthermore, we do not require the measurement errors η_k and η_l to be uncorrelated for $k \neq l$. A weaker mean independence restriction is assumed by Hausman et al. (1991) and Schennach (2004). However, in order to identify the parameters of interest, these authors also assumed the existence of an additional noisy measure of the true unobserved regressor. The method developed here does not rely on the existence of auxiliary data sets. It does depend on the assumptions of iso - elastic utility, multiplicative specification of habits, and multiplicative measurement error. However, as discussed in the introduction, there are good reasons for imposing these restrictions when investigating the

[7] Chen and Ludvigson (2009) and Smith and Zhang (2007) undertake the empirical estimation of the consumption model with habit formation using nonlinear exact estimation. But the authors deal with aggregate consumption data where measurement errors issue are not a significant concern.

existence of habit formation in micro consumption data. Furthermore, the method developed in this paper for accounting for measurement errors can be applied to a larger class of moment conditions that take a similar form.

Define $g_{it}^o = c_{it}^o / c_{it-1}^o$ and $v_{it} = \tilde{\eta}_{it}/\tilde{\eta}_{it-1} = \eta_{it}/\eta_{it-1}$, so that $g_{it}^o = g_{it} v_{it}$. Then we have the following theorem

Theorem 3.2. *Suppose Assumption 3.1 is satisfied, then there exists positive constants $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$, such that equation (3.1) implies*

$$E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}^o}\left(\frac{g_{it+1}^o}{g_{it}^{o\alpha}}\right)^{1-\gamma}\left(\mathcal{A}_1^{-1}-\alpha\beta\mathcal{A}_2^{-1}\varphi_{it+2}\left(\frac{g_{it+2}^o}{g_{it+1}^{o\alpha}}\right)^{1-\gamma}\right)-\right.$$
$$\left.\left(1-\alpha\beta\mathcal{A}_3^{-1}\varphi_{it+1}\left(\frac{g_{it+1}^o}{g_{it}^{o\alpha}}\right)^{1-\gamma}\right)\middle| z_{it}^o\right]=0, \tag{3.2}$$

where z_{it}^o is a q-dimensional observable subset of z_{it}.

The proof of Theorem 3.2 is found in Appendix A. To illustrate our method, consider the first term in in equation (3.1) with observed consumption growth substituted for true consumption growth. Then, under Assumption 3.1, we have that

$$\begin{aligned}E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}^o}\left(\frac{g_{it+1}^o}{g_{it}^{o\alpha}}\right)^{1-\gamma}\middle| z_{it}^o\right] &= E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\frac{1}{v_{it+1}}\left(\frac{v_{it+1}}{v_{it}^{\alpha}}\right)^{1-\gamma}\middle| z_{it}^o\right]\\ &= E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\mathcal{A}_3\middle| z_{it}^o\right],\end{aligned}$$

where the second equality is obtained from the law of iterated expectations and Assumption 3.1. Thus,

$$E\left[\beta\mathcal{A}_1^{-1}(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}^o}\left(\frac{g_{it+1}^o}{g_{it}^{o\alpha}}\right)^{1-\gamma}\middle| z_{it}^o\right] = E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\middle| z_{it}^o\right].$$

Performing this process on the other two terms in equation (3.1) obtains equation (3.2). Note that, given the structure of equation (3.1), if lagged income (in the taste shifters) is contaminated with classical measurement errors then the latter are absorbed in the $\mathcal{A}$s.

It is of interest to evaluate the variation in observed food consumption due to measurement errors. The variance of measurement errors is not identified without additional assumptions. To this end, we will make the following functional form assumption as an alternative specification.

Assumption 3.3. *Conditional on μ_i, measurement errors in consumption are serially independent and log-normally distributed:*

$$\ln \eta_{it} | \mu_i \sim N(0, \sigma^2). \tag{3.3}$$

Under this additional assumption, we have that

$$\begin{aligned}
\mathcal{A}_1 &= \exp\{\sigma^2 \left(\alpha^2(1-\gamma)^2 + \gamma^2 - \alpha\gamma(1-\gamma)\right)\} \\
\mathcal{A}_2 &= \exp\{\sigma^2 \left(\alpha^2(1-\gamma)^2 + \gamma^2 + (1-\gamma)(1+\alpha)\right)\}, \quad \text{and} \\
\mathcal{A}_3 &= \exp\{\sigma^2 \left((1+\alpha+\alpha^2)(1-\gamma)^2\right)\}.
\end{aligned} \tag{3.4}$$

The details of the derivation can be found in Appendix A. As discussed in the previous paragraph, the method proposed in this paper is also robust to classical measurement errors in income without further assumptions. To investigate the degree of measurement errors in income and its effect on the parameters of interest, assume the following model that relates observed income y_{it}^o to unobserved true income y_{it}

$$y_{it}^o = y_{it} + \tilde{\upsilon}_{it}, \tag{3.5}$$

where $\tilde{\upsilon}_{it} = \zeta_i + \upsilon_{it}$, and ζ_i is the household-specific measurement error effect in observed income.

Assumption 3.4. *Conditional on ζ_i, measurement errors in income are serially independent, independent from the time vector of consumption, the taste shifters, the information set, the interest rate, income, measurement errors in consumption, and normally distributed:*

$$\upsilon_{it}|\zeta_i \sim N(0, \varsigma^2). \tag{3.6}$$

Under this additional assumption,

$$\begin{aligned}
\mathcal{A}_1 &= \exp\{\varsigma^2 + \sigma^2\left(\alpha^2(1-\gamma)^2 + \gamma^2 - \alpha\gamma(1-\gamma)\right)\} \\
\mathcal{A}_2 &= \exp\{\varsigma^2 + \sigma^2\left(\alpha^2(1-\gamma)^2 + \gamma^2 + (1-\gamma)(1+\alpha)\right)\}, \quad \text{and} \\
\mathcal{A}_3 &= \exp\{\varsigma^2 + \sigma^2\left((1+\alpha+\alpha^2)(1-\gamma)^2\right)\}.
\end{aligned} \tag{3.7}$$

3.3 Identification

This section investigates identification of the parameters from the moment condition in equation (3.2). We first consider identification of the parameters of the model without imposing Assumption 3.3. To this end, let $\kappa_1 = \mathcal{A}_2/\mathcal{A}_1$, $\kappa_2 = \mathcal{A}_2$, and $\kappa_3 = \mathcal{A}_2/\mathcal{A}_3$. Then, noting that $\mathcal{A}_2 > 0$, equation (3.2) can be rewritten as

$$\begin{aligned}
E\Bigg[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g^o_{it+1}}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\left(\kappa_1 - \alpha\beta\varphi_{it+2}\left(\frac{g^o_{it+2}}{g^{o\alpha}_{it+1}}\right)^{1-\gamma}\right) - \\
\left(\kappa_2 - \alpha\beta\kappa_3\varphi_{it+1}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\right)\Bigg| z^o_{it}\Bigg] \;=\; 0,
\end{aligned} \tag{3.8}$$

Define $x^o_{it+2} = \left(g^o_{it+2},\ g^o_{it+1},\ g^o_{it},\ r_{it+1},\ \Delta w_{it+2}, \Delta w_{it+1}\right)$, $\kappa = (\kappa_1, \kappa_2, \kappa_3)$, $\theta = (\alpha, \beta, \gamma, \kappa, \delta)'$, and $\Delta_2 w_{it} = w_{it} - w_{it-2}$. The following conditions are sufficient for the identification of θ.

Assumption 3.5. *1. For at least one t in $\{4, \cdots, T\}$, the conditional distribution of x^o_{it} given z^o_{it-2} is complete and is not contained in any proper linear subspace of $\Re^6$.*

2. $\delta_1 = 1$.

See Newey and Powell (2003) for discussions on the first part of Assumption 3.5.1. This completeness restriction is also imposed in Chen and Ludvigson (2009) to prove identification of the parameters characterizing their asset pricing model. As T increases, it becomes easier to find at least one period for which this condition is satisfied. The second part of Assumption 3.5.1 is a full rank assumption. One consequence of this restriction is that a constant cannot be included in w_{it}. Furthermore, a variable that changes by a constant amount, such as age, may not be included in w_{it}. Assumption 3.5.2 eliminates the trivial solution problem that plagues estimation of consumption Euler equations. In our case, notice that by setting $\delta = 0$, $\gamma = 1$, and $\alpha\beta = \kappa_1 = \kappa_2/\kappa_3$, the term inside the expectation on the right hand side of equation (3.8) becomes identically zero. Setting $\delta_1 = 1$ eliminates this trivial solution. However, this assumption does require a priori knowledge of the sign of δ_1. In the next section we discuss the choice of $w_{it,1}$. Let θ_0 denote the true parameter vector. The proof of the following theorem is presented in Appendix B

Theorem 3.6. *Consider equation (3.8) and suppose that Assumption 3.5 holds. Then α, γ, δ, $\kappa_1\beta$, κ_2, and $\kappa_3\beta$ are uniquely identified. Furthermore, if $\alpha > 0$, θ_0 is uniquely identified.*

The intuition behind the identification of the parameters is as follows. If there is no habit formation ($\alpha = 0$), then variation in the interest rates, along with variation in consumption growths in periods t and $t+1$ identify γ, δ, $\kappa_1\beta$, κ_2, $\kappa_3\beta$, and α (at zero). This result is similar to Alan et al. (2009). Note that if there are no measurement errors in consumption, then $\kappa_1 = \kappa_2 = \kappa_3 = 1$ so that, as is standard, β is identified. On the other hand, if there habit formation exists ($\alpha > 0$), then the consumption growth rate in period $t+2$ becomes relevant in equation (3.8). Variation in consumption growth rate in period $t+2$ identifies β, and hence also $(\kappa_1, \kappa_2, \kappa_3)$. The role of consumption growth in period $t+2$ in identifying β is similar to its role in the second estimator of Alan et al. (2009). Therefore, to summarize, γ, α, δ, $\kappa_1\beta$, κ_2, and $\kappa_3\beta$ are identified whether or not $\alpha = 0$. Furthermore, if $\alpha > 0$, then β and (κ_1, κ_3) are also identified.

If Assumptions 3.3 and 3.6 are also imposed, then σ^2 and ς^2 are identified from the resulting structure of κ. It is important to note that these variances should be considered a lower bound on the amount of noise present in observed consumption and income, because the additional variation contributed by the household-specific effect are not accounted for.

We can therefore estimate the unknown structural parameters of interest using equation (3.2) as a conditional moment of the form

$$E\left[\rho(x^o_{it+2}, \theta_0) | z^o_{it}\right] = 0, \tag{3.9}$$

Define the q-dimensional vector $m_{it}(\theta) := m(x^o_{it+2}, z^o_{it}, \theta) := z^{o'}_{it}\rho(x^o_{it+2}, \theta)$ and the corresponding $q(T-4)$-dimensional moment vector $m_i(\theta) = m(x^o_i, z^o_i, \theta) := (m'_{i3}(\theta), \cdots, m'_{iT-2}(\theta))'$ Then equation (3.9) implies that

$$m(\theta_0) = E[m_i(\theta_0)] = 0. \tag{3.10}$$

Let $\hat{m}(\theta) := \sum_{i=1}^N m_i(\theta)/N$ and $\hat{\Omega}(\theta) := \sum_{i=1}^N m_i(\theta)m'_i(\theta)/N$. Then, our estimator for the parameters of interest is defined by

$$\hat{\theta} = \arg\min_{\theta\in\Theta} \hat{m}'(\theta)\hat{\Omega}(\theta)^+\hat{m}(\theta), \tag{3.11}$$

where $\hat{\Omega}(\theta)^+$ is the generalized inverse of $\hat{\Omega}(\theta)$. Two remarks are in order.

Whereas identification was discussed in terms of conditional moment restrictions, the estimator is defined using the unconditional moment restrictions (equation 3.10) implied by their conditional counterparts. This is the prevailing approach taken in applied work. However, in our case it is important to know whether the identification strategy implemented in this paper, using conditional moment restrictions, still applies under the unconditional moment restriction. Indeed, if equation (3.8) was written in the form of an unconditional moment then identification of the model's parameters is maintained if the completeness assumption is instead imposed on the joint distribution $f(x_{it}^{o}, z_{it-2}^{o})$.[8]

As suggested by Hansen et al. (1996), we apply continuous updating GMM (CUGMM) to obtain estimates of the structural parameters. Although CUGMM is known to be somewhat difficult to implement, it has advantages that are pertinent. As stated by Hansen et al., CUGMM alleviates the problem of weak identification of parameters that is common in estimating Euler equations. However this doesn't stop the estimator from approaching the trivial solution, in which case our experience indicates that the estimator becomes unstable. As discussed in the identification section, we eliminate this instability by imposing the restriction that $\delta_1 = 1$, thus eliminating the choice of $\delta = 0$. However, this requires prior knowledge about which variable in the taste shifter should have a positive coefficient. For this, we make use of the stylized fact that the marginal utility of consumption is decreasing in income. As can be seen from equation (2.5), setting the coefficient on lagged income equal to one imposes this restriction. This restriction significantly increases the stability of the estimation algorithm up to a point where one might be tempted to replace CUGMM with two-step GMM. However, with approximately 50 moments (5 instruments and 10 periods) used in the estimation, CUGMM is also attractive in that it tends to reduce the bias found in efficient two-step GMM with many moments (see Newey and Smith, 2000).[9,10]

[8]Similar to Chen and Ludvigson (2009), an alternative method of estimating the Euler equation is to directly estimate the conditional expectations by employing the sieve minimum distance (SMD) technique (see also Ai and Chen, 2003). However, this method is computationally more intensive, and less is known about testing model validity in the SMD framework.

[9]Application of one- and two-step GMM estimators to our model confirmed this result, as the estimated parameters (especially γ) were outside reasonable ranges.

[10]In our habit formation model with no measurement errors, the period t moment is correlated with the period

4 Monte Carlo Experiment

In this section we investigate the finite sample performance of the approximated log-linearized habit formation model and the procedure developed in this paper. The results from implementing the approximated estimator using the simulated data show that the habit formation parameter estimate has substantial downward bias. On the other hand, the simulation exercise shows that the estimator developed in this paper performs well in recovering the parameters of interest. Also, the results indicate that ignoring measurement errors in food consumption results in a downward bias in the estimate of the habit formation parameter.

We conduct a Monte Carlo simulation where the life-cycle model presented in Section 2 is solved under labor income and interest rate uncertainty.[11] The structural parameter values are set as follows: $\gamma = 5$, $\alpha = 0.85$, $\beta = 0.95$. The interest rate series is a stationary AR(1) process with a mean of 0.05 and autoregressive coefficient of 0.6. We solve the model for 40 periods, however in estimation we only use the 10 middle periods so that the length of the artificial panel matches the one used in the empirical analysis. Additionally, due to this trimming, starting and ending effects of the artificial consumption series are not an issue. Consumption paths are simulated to obtain 100 samples of 1700, individuals observed over 10 periods. Next, the simulated consumption data is contaminated with measurement errors drawn independently over individuals and time from a log-normal distribution with variance equal to 75% of the variance in consumption.

With the simulated data in hand we investigate the performance of GMM estimation of the linear approximation models developed in Hayashi (1985), Muellbauer (1988), and Dynan (2000).

t+1, and t+2 moments, in which case we have $5 \times 3 = 15$ moment conditions. If measurement errors are allowed to be arbitrarily correlated over time, then the period t moment is potentially correlated with the moments of all other periods, in which case we have $5 \times 10 = 50$ moment conditions.

[11]The details of the solution and simulation methods are standard for the intertemporal utility optimization framework and available from the authors upon request.

To derive the estimator for the additive habit model, it is assumed that: (i) interest rates do not vary across individuals or over time; (ii) individuals live for infinite period; and (iii) $\Delta\ln(c_t - \alpha c_{t-1}) \approx \Delta\ln(c_t) - \alpha\Delta\ln(c_{t-1})$. We derive a comparable estimator for which the first two assumptions are maintained. As shown in Muellbauer (1988), the third assumption requires that consumption does not vary significantly over time. In the multiplicative habit model, this and the first two assumptions imply that IV estimation of

$$\Delta\ln(c^o_{it}) = \beta_0 + \alpha\Delta\ln(c^o_{it-1}) + \beta_1 age_t + \beta_2 age^2 + \varepsilon_t \tag{4.1}$$

should yield the result the result $\alpha = 0.85$. The instruments for $\Delta\ln(c^o_{it-1})$ are the first two lags of income growth and the lagged interest rate.

Two sets of results from the Monte Carlo investigation of this estimation method are reported in Table 1. The first two columns are the results in the absence of consumption measurement errors, and the last two are in its presence. The results show significant downwards bias in the estimate of α even without consumption measurement errors. These results suggest that the assumptions made to obtain equation (4.1) are substantial. The bias is more severe when consumption is measured with errors.

Even if estimation of equation (4.1) was successful in recovering α, it does not identify β and γ, which are also needed to investigate the IES and RRA. Direct estimation of equation (3.2) provides estimates of all the parameters needed to analyze the IES and RRA. Table 2 presents results from the Monte Carlo investigation of the estimator developed in the previous section. Column (1) gives the true values of the preference parameters that we try to recover using the proposed estimator. Column (2) shows that the estimator performs well in the absence of consumption measurement errors. The results also show that the estimator performs well when the distribution of measurement

Table 1: Estimation of equation (4.1) using the simulated data

Parameters	No ME		Nonparametric ME	
	(1)	(2)	(3)	(4)
β_0 (Constant)	0.042	-0.005	0.175	0.094
	[0.041]	[-0.006]	[0.175]	[0.094]
	(0.040)	(0.039)	(0.047)	(0.048)
α ($\Delta\ln(c^o_{it-1})$)	0.170	0.197	0.106	0.163
	[0.171]	[0.197]	[0.105]	[0.161]
	(0.015)	(0.015)	(0.020)	(0.021)
β_1 (Age)	0.0001	-0.0004	-0.013	-0.012
	[0.0001]	[-0.0003]	[-0.013]	[-0.011]
	(0.004)	(0.004)	(0.004)	(0.005)
β_2 (Age^2/1000)	-0.091	-0.077	0.204	0.194
	[-0.097]	[0.082]	[0.192]	[0.180]
	(0.101)	(0.099)	(0.121)	(0.120)
β_3 ($\ln(1+r_t)$)	—	1.121	—	1.540
		[1.112]		[1.525]
		(0.149)		(0.243)

Instrument set includes the first two lags of income growth and lagged interest rate. In columns 2 and 4, $\ln(1+r_t)$ is treated as endogenous. Standard errors in parentheses.

errors is known to be log-normal (column 3), and when the distribution of measurement errors is unknown (column 4). Column (5) shows that not accounting for measurement errors result in upward bias in γ and downward bias in α and β.

Table 2: Estimation of the Euler equation with habit formation, using simulated data[1]

Parameters	Truth (1)	No ME (2)	Log-normal ME (3)	Nonparametric ME (4)	Ignoring ME (5)
γ	5.00	4.93	5.73	5.16	13.22
		[4.89]	[4.98]	[5.00]	[13.32]
		(0.33)	(2.65)	(1.03)	(2.03)
β	0.95	0.95	0.94	0.95	0.74
		[0.95]	[0.95]	[0.95]	[0.74]
		(0.01)	(0.07)	(0.01)	(0.05)
α	0.85	0.85	0.85	0.85	0.55
		[0.85]	[0.85]	[0.85]	[0.55]
		(0.01)	(0.03)	(0.01)	(0.05)
σ^2	0.04		0.04		
			[0.03]		
			(0.04)		

[1] In estimation we reduce the time dimension of the artificial data panel to 10 years. Instrument set includes current and past interest rates and current income. Standard errors are in parentheses. Medians are in square brackets.

5 Data

Data on food consumption, as well as income and demographic characteristics of individuals and households are available from the Panel Study of Income Dynamics (PSID). Although it is the longest panel study, and one of the most comprehensive sources of information for studying life-cycle processes and poverty and welfare dynamics, its use for studying consumption involves one drawback: consumption data are available only for food. Fortunately, data on consumption of food as a perishable good are particularly suitable for testing whether this category of consumption can be habit-forming. The annual frequency of observation is also advantageous. As argued in Dynan (2000), if there is any effect of durability in food consumption, it is not likely to last more than a few months.

The main consumption sample that we use consists of data from 1974 through 1987. Consumption of households consists of expenditures on food consumed at home, away from home, and the value of food stamps. Data on food consumed at home and the value of food stamps are deflated using the consumer price index (CPI) for food at home. Data on food consumed away from home are deflated using the CPI deflator for food away from home. All CPI data are taken from the consumer price index releases of the Bureau of Labor Statistics. Food consumption data are deflated according to the month and year when the interview occurred, while food stamps and data on income are deflated using the CPI for the end of the year before the interview was conducted. In addition, total consumption expenditures are adjusted by the size of household.

In the sample we keep only the households who report positive savings over the sample period. This is done to exclude liquidity constrained households for whom the Euler equation (2.4) does not hold. We exclude households whose marital status changed or whose head was younger than 22 or older than 65 over the period of estimation. We also exclude observations for which the consumption growth rate was higher than 300% and lower than 33%. It is likely that the extreme outliers in consumption growth rate that we observe in the untrimmed data are due to measurement errors. Thus, the estimated magnitude of the variance of the measurement errors in consumption is to be considered a lower bound after this data trimming. Household characteristics used in estimation as taste shifters include past income, family size and age of the head of household.

As in Shapiro (1984), Runkle (1991), and similar studies, we construct the household-specific real after-tax interest rate as $r_{it+1} = R_t(1 - \tau_{it+1}) - \pi_{t+1}$, where R_t is the average 12-month Treasury bill for the first half of the preceding year, τ_{it+1} is the household marginal tax rate as reported in the PSID, and π_{t+1} is the CPI deflator for the period of the interview.

The estimation of the moment condition (equation 3.8) and of most of its modifications for robustness checks requires data on consumption expenditures for four consecutive years for each orthogonality condition. With the restrictions on data described above, we have an unbalanced panel on 1,754 liquidity unconstrained households covering ten years from 1976 through 1985.

6 Empirical Results

In this section we address several issues while discussing the results obtained from the estimation. The main conclusion from the results are that: (i) habit formation plays an important role in explaining household food consumption patterns; and (ii) not accounting for measurement errors in observed consumption (and income) result in a downward bias in the estimates of the habit formation parameter and the discount factor.

Table 3 presents the parameter estimates from various specifications of the model. In all specifications, we include as taste shifters the lagged income, current family size, and squared age of the

Table 3: Estimation of the Euler equation with habit formation[1]

Parameters	Nonparametric ME Internal habit	Nonparametric ME External habit	Parametric ME cons	Parametric ME cons, inc	Ignoring ME	Approx. GMM
	(1)	(2)	(3)	(4)	(5)	(6)
γ	4.950	7.960	3.029	3.833	5.075	
	(0.099)	(0.120)	(0.366)	(0.077)	(0.444)	
β	0.929	0.941	0.956	0.981	0.305	
	(0.043)	(0.061)	(0.030)	(0.014)	(0.062)	
α	0.807	0.500	0.799	0.848	0.205	-0.190
	(0.028)	(0.010)	(0.115)	(0.078)	(0.114)	(0.120)
a		0.353				
		(0.016)				
σ^2			0.036	0.034		
			(0.002)	(0.002)		
ς^2				0.031		
				(0.020)		
J statistic	25.6	29.4	30.6	23.67	37.6	15.5[2]
p value	0.978	0.911	0.936	0.993	0.774	0.000[2]

[1] Number of time periods T = 10, number of households $N = 1,754$. The instrument set includes current and past Treasury bill rates, household size, age of household head, and a constant. Standard errors in parenthesis.
[2] F-test for excluded instruments (dummies for lagged income and hours growth rates, and dummy for whether head of the household lost job in previous period) and corresponding p-value reported. Seven households were lost due to missing observations on the dummy for whether the head of the household lost job in previous period.

head of the household. Recall from the last paragraph of Section 3, we set the coefficient on lagged income equal in order to 1 as to impose the restriction that the marginal utility of food consumption is decreasing in income. Column (1) shows results for the baseline model. Recall that habit formation exists in the multiplicative model if $\gamma > 1$ and $\alpha > 0$. The results show that the estimate of γ is significantly greater than one (4.95) and the estimate of α is significantly greater than zero

(0.81). Therefore, the estimates of the baseline model support the existence of habit formation in individual food consumption.

The baseline model is specified to be consistent only with habit being internal to the household: the household's period-specific utility depends only on its past consumption and not the past consumption of others. As an extension, we allow for external habits in household consumption by augmenting the definition of consumption services as follows:

$$\tilde{c}_{it} = \frac{c_{it}}{c_{it-1}^{\alpha} C_{it-1}^{a}}, \tag{6.1}$$

where C_{it} is period t average consumption of household i's income group, and $0 \leq a \leq 1$ measures the strength of external habits. This extension allows for the household's period-specific utility to also depend on past aggregate consumption of the household's income group. Aggregate consumption is constructed for 4 different income groups, with roughly the same number of households in each. We assume that measurement errors in aggregate consumption are averaged out. Then the external habit counterpart of Equation (3.5) is:

$$E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}^{o}}\left(\frac{g_{it+1}^{o}}{g_{it}^{o\alpha}G_{it}^{a}}\right)^{1-\gamma}\left(\kappa_1 - \alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}^{o}}{g_{it+1}^{o\alpha}G_{it+1}^{a}}\right)^{1-\gamma}\right) - \right.$$
$$\left.\left(\kappa_2 - \alpha\beta\kappa_3\varphi_{it+1}\left(\frac{g_{it+1}^{o}}{g_{it}^{o\alpha}G_{it}^{a}}\right)^{1-\gamma}\right)\middle| z_{it}^{o}\right] \ = \ 0,$$

where $G_{it} = C_{it}/C_{it-1}$.

Column (2) of Table 3 indicates that, in addition to internal habits, external habits are significant in explaining household food consumption patterns. However, the strength of external habit is significantly smaller that the strength of internal habit, with a estimated to be 0.35 while α is estimated to be 0.50. Therefore, while internal habit formation has the dominant effect, external habit formation also plays an important role in explaining consumption patterns.

Columns (3) and (4) report the estimation results when it is assumed that the distribution of measurement errors in consumption is log-normal and there are no measurement errors in income (column 3), and where it is also assumed that the distribution of measurement errors in income is normal (column 4). Both estimates support the existence of habit formation in food consumption. However, relative to the baseline model, the estimates of γ are significantly smaller and the point estimates of β are slightly larger.

Column (5) of Table 3 presents the estimation results where measurement errors in consumption and income are not accounted for. Consistent with the Monte Carlo exercise, we find significant downwards bias in the habit formation parameter and the discount factor. The habit formation parameter is only modestly significantly different from zero. Thus, not accounting for measurement errors biases the results against finding habit formation in food consumption.

Column (6) presents the estimate of α from the linearized model in equation (4.1). Following Dynan (2000), the instruments for $\Delta \ln c_{it-1}^{o}$ include dummies for the ranges of lagged growth in income and hours worked, as well as a dummy for whether the head of the household became involuntarily unemployed in the previous period. The estimate of α is not significantly different from zero at the 5% level of significant. Hence, as found in the Monte Carlo exercise, estimates from the linearized model are biased against finding habit formation in food consumption.

Two additional potential concerns about the estimation are worth addressing. The first is that aggregate shocks in the expectation errors are likely to bias our estimates if they are not accounted for. The model is also estimated allowing for aggregate effects in the expectations errors. We find no significant changes to the results reported in Table 3. These results are not reported in this paper, but can be found in the earlier version and are available from the authors upon request. Second, Meghir and Weber (1996) suggest that the finding of habit formation in food consumption may be explained by nonseparabilites in preferences over food and other consumption goods. Carrasco et al. (2005) find that this result is largely due the presence of time-invariant unobserved heterogeneity that Meghir and Weber were unable to control for due to the small length of the panel used in their estimation. Our estimation method does control for time-invariant heterogeneity. Furthermore, if nonseparabilites in preferences over food and other consumption goods represent a significant misspecification in our model, then it is likely that this misspecification would be detected in the J test, as with the specification that ignores measurement errors.

7 Intertemporal Elasticity of Substitution and Risk Aversion

In this section, we analyze the intertemporal elasticity of substitution (IES) and the relative risk aversion (RRA) that are implied by the estimates obtained in the previous section. In the presence of habit formation, the IES and the RRA vary across individuals and over time. Specifically, Appendices C and D show that the inverse IES and the RRA take the following form:

$$\frac{1}{IES_{it}} = \gamma - \frac{\alpha\beta(1-\gamma)\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}}{1-\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}} - \frac{\alpha^2\beta(1-\gamma)\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}}{1-\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}}, \tag{7.1}$$

$$RRA_{it} = \frac{\gamma-\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma} - \alpha^2\beta(1-\gamma)\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}}{1-\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}}. \tag{7.2}$$

Three important facts about the IES and RRA are made clear by observing equations (7.1) and (7.2). First, the inverse IES and the RRA are higher for habit forming consumers than for non-habit forming consumers. Second, the model with habit formation implies heterogenous IES and RRA. Recent studies that allow for heterogeneity in risk aversion find significant variation in risk aversion across different groups of individuals.[12] Third, the habit formation model breaks the tight inverse relationship between the IES and the RRA of iso-elastic preference specification. These observations imply that the habit formation model is able to explain varying IES and RRA across groups of individuals in ways that the iso-elasticity models cannot.

Because true consumption is not observed, the IES and RRA are generally not observed. Furthermore, the conditional expectations of the inverse IES and the RRA do not conform to the transformation used to derive the estimator because their functional forms do not satisfy the conditions used to separate true consumption from measurement errors. Therefore, the expectations of the (inverse) IES and the RRA are in general not directly recoverable from equations (7.1) and (7.2). However, it is possible to construct bounds for the conditional expectation of these quantities given the set of instruments z. The proof of the following proposition is given in Appendix C.

Proposition 7.1. *Suppose Assumption 3.1 holds. Then*

$$\begin{aligned}\frac{1}{\gamma} \geq E\left[IES_{it}|z_{it}\right] \geq \; & \Bigg(\gamma - (1-\gamma)\textstyle\sum_{j=1}^{\infty} E\left[\left(\mathcal{A}_3^{-1}\alpha\beta\varphi_{it+1}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\right)^j |z_{it}\right] \\ & -\alpha(1-\gamma)\textstyle\sum_{j=1}^{\infty} E\left[\left(\mathcal{A}_3^{-1}\alpha\beta\varphi_{it+2}\left(\frac{g^o_{it+2}}{g^{o\alpha}_{it+1}}\right)^{1-\gamma}\right)^j |z_{it}\right]\Bigg)^{-1},\end{aligned}$$

and

$$\gamma \leq E\left[RRA_{it}|z_{it}\right] \leq \gamma - (1+\alpha)(1-\gamma)\sum_{j=1}^{\infty} E\left[\left(\mathcal{A}_3^{-1}\alpha\beta\varphi_{it+1}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\right)^j |z_{it}\right],$$

with strict inequalities if $\alpha > 0$, $\beta > 0$ *and* $\gamma > 1$.

Proposition 7.1 can be used to construct bounds for the unconditional expectation of the IES and the RRA.

[12] Crossley and Low (2011) find heterogeneity in the IES across groups of goods and services.

As shown in the next proposition, the assumption that measurement errors are independent and log-normally distributed results in point identification of the expectation of the RRA. However, the expectation of the IES is still only partially identified, but typically with narrowed bounds relative to the more general case above.

Proposition 7.2. *Suppose Assumptions 3.1 and 3.3 hold. Then*

$$\frac{1}{\gamma} \geq E\left[IES_{it}|z_{it}\right] \geq \left(\gamma - (1-\gamma)\sum_{j=1}^{\infty} E\left[\mathcal{A}_3^{-j^2}\left(\alpha\beta\varphi_{it+1}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\right)^j |z_{it}\right] -\alpha(1-\gamma)\sum_{j=1}^{\infty} E\left[\mathcal{A}_3^{-j^2}\left(\alpha\beta\varphi_{it+2}\left(\frac{g^o_{it+2}}{g^{o\alpha}_{it+1}}\right)^{1-\gamma}\right)^j |z_{it}\right]\right)^{-1},$$

with strict inequality if $\alpha > 0$*,* $\beta > 0$ *and* $\gamma > 1$*, and*

$$E\left[RRA_{it}|z_{it}\right] = \gamma - (1+\alpha)(1-\gamma)\sum_{j=1}^{\infty} E\left[\mathcal{A}_3^{-j^2}\left(\alpha\beta\varphi_{it+1}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\right)^j |z_{it}\right],$$

where $\mathcal{A}_3 = \exp\{\sigma^2\left((1+\alpha+\alpha^2)(1-\gamma)^2\right)\}$.

To see that the bounds on $E\left[IES_{it}|z_{it}\right]$ as defined in Proposition 7.1 are typically wider than defined in Proposition 7.2 under the independent log-normal assumption, notice that $\mathcal{A}_3 > 1$ so that for $j > 1$, $\mathcal{A}_3^{j^2} > \mathcal{A}_3^j$. Each additional term in the infinite sums in Proposition 7.2 is smaller than the corresponding term in Proposition 7.1.

Given the parameter estimates of the model, the bounds on the IES and RRA can be approximated with high precision by replacing the infinite sum with a finite approximation. Consistency of these estimators of the bounds would depend on allowing the order of approximation to increase with the sample size. The results of Ai and Chen (2003) can be used to prove this consistency conjecture. However, this is beyond the scope of the current paper and is left to future work. We

Table 4: 95% confidence intervals for the IES, inverse IES and RRA

	Computed using the parameter estimates in Table 3, column: (1)	(2)	(3)	(4)
IES	[0.083, 0.193]	[0.071, 0.124]	[0.104, 0.286]	[0.086, 0.242]
RRA	[4.991, 13.226]	[7.961, 15.486]	[3.118, 8.412]	[3.862, 9.645]

employ a fifth-order approximation of these infinite sums. With this in hand, asymptotically valid confidence sets can be defined for these bounds. Construction of these confidence sets is found in Horowitz and Manski (2000) and Imbens and Manski (2004).

Table 4 presents 95% confidence intervals for the expectation of the IES and the RRA with estimated parameters taken from selected specifications in Table 3. 20 bootstrap draws from the estimated asymptotic distribution of the estimated parameters are used to compute the standard error of these bounds. The columns of Table 4 are labeled (1), (2), (3), and (4) to correspond to the columns of Table 3.

The estimated bounds for the IES support typical findings in the literature on the estimation of wealth and consumption. In the habit formation framework, Naik and Moore (1996) report an IES close to our estimates. A similar range for the IES is found across households or certain cohorts of individuals (see Barsky et al., 1997). However, larger values for the IES are also reported in the literature.[13] It is important to note that as we investigate household food consumption, the estimated bounds for the IES seem to be reasonable: the consumption of food is likely to be relatively inelastic.

[13] See, for example, Attanasio and Weber (1993), Atkeson and Ogaki (1996), Vissing-Jorgensen (2002), and Crossley and Low (2011).

As expected, the estimated bounds on the RRA are higher than the usual range found for consumption models without habit formation. However, recent developments in empirical estimates of risk aversion find estimates well inside the suggested bounds. Alan and Browning (2010) provide a discussion on the recent literature and show estimation results supporting higher values of the risk aversion parameter. In addition, higher values and greater dispersion are supported by the survey-based studies of Barsky et al. (1997), Eisenhauer and Ventura (2003), and Guiso and Paiella (2006).

To investigate the existence and significance of heterogeneity in the IES and the RRA, we regress the individual time-specific IES and RRA, calculated using observed consumption, on a set of regressors. Because the additional noise introduced into the dependent variable by measurement errors is independent from the other variables in the regression function, we expect the estimated coefficients to be biased towards zero. Therefore, the regression results should be interpreted as being biased against detecting statistically significant explainable heterogeneity in the IES and the RRA.

Table 5 reports the results from the regression of observed IES and RRA on a constant, lagged income, lagged income squared, a dummy for high school graduate (HG), a dummy for college graduate (CG), age, and age squared. Observed IES and RRA are computed using the estimated parameters from columns (1) - (4) of Table 3, and the columns labeled (1), (2), (3), and (4) of Table 5 correspond to the columns of Table 3.

The regression results indicate that the IES is decreasing and convex in both age and income, while the RRA is increasing and concave in age and income. We do not find that education is a significant determinant of the IES and RRA. These results are consistent across the different specifications, except for specifications (2) and (3), where we do not find significant effects of income on the RRA.

Table 5: Regression of estimated RRA and IES on income and age

	RRA				IES			
	Parameter estimates in Table 3, column:				Parameter estimates in Table 3, column:			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Constant	5.976	10.05	4.982	5.371	0.121	0.097	0.186	0.156
	(2.077)	(2.062)	(0.630)	(0.899)	(0.010)	(0.007)	(0.008)	(0.008)
Income_{t-1}	5.610	2.352	0.733	2.862	-0.041	-0.016	-0.049	-0.047
	(2.974)	(2.956)	(0.927)	(1.328)	(0.015)	(0.009)	(0.011)	(0.012)
Income^2_{t-1}	-9.533	-3.294	-1.178	-3.799	0.058	0.029	0.071	0.062
	(3.481)	(3.457)	(1.075)	(1.549)	(0.017)	(0.011)	(0.013)	(0.014)
HG_t	-0.024	-0.289	0.059	-0.096	0.002	0.001	0.0007	0.0005
	(0.356)	(0.356)	(0.112)	(0.159)	(0.002)	(0.001)	(0.0013)	(0.0014)
CG_t	0.141	-0.459	0.167	0.200	0.002	0.0018	-0.0003	-0.0005
	(0.435)	(0.430)	(0.133)	(0.190)	(0.002)	(0.0014)	(0.0016)	(0.0017)
Age_t	0.275	0.225	0.104	0.123	-0.0013	-0.0010	-0.0018	-0.0010
	(0.100)	(0.100)	(0.030)	(0.044)	(0.0004)	(0.0003)	(0.0004)	(0.0004)
Age^2_t	-0.0029	-0.0025	-0.0010	-0.0012	0.00001	0.00001	0.00002	0.00002
	(0.0011)	(0.0011)	(0.0003)	(0.0005)	(0.000006)	(0.000001)	(0.000004)	(0.000004)
R^2	0.0078	0.0040	0.0047	0.0054	0.0092	0.0079	0.0143	0.0152

HG is a dummy for hight school graduates, CG is a dummy for college graduates. Income_{t-1} is divided by 100,000 and Age_t is divided by 100.

In terms of individual actions, the results are interpreted as follows. Given income and education, the intertemporal allocation of food consumption by households with older heads is less responsive to intertemporal changes in food prices. Also, given income and education, households with older heads are more likely to dedicate a larger percentage of their income to precautionary savings than their younger counterparts.

Given education and age, households with moderate income (roughly between \$27,000 and \$39,000) are less responsive to intertemporal changes in food prices, and are more likely to save for precautionary motives.

8 Conclusion

In this paper, we exploit the property of habit formation preferences to generate coefficients of relative risk aversion and intertemporal elasticity of substitution that vary across households and over time in order to analyze the degree of heterogeneity in these economic quantities. Our analysis hinges on the evidence of habit formation in consumption, which at the level of microdata is shown to be mixed. We argue that previous micro studies that investigate habit formation using standard preferences impose arguably strong assumptions in order to obtain an estimating equation. The misspecifications that result from these assumptions are likely to result in significant biases in the estimates. This intuition is confirmed in our simulation exercise.

This paper develops a new exact nonlinear GMM estimator that accounts for measurement errors without the need for parametric assumptions on their distribution. We find that habit formation is an important determinant of food consumption patterns. Not accounting for measurement errors biases the estimates towards rejecting the existence of habit formation. The method to control for measurement error does rely on the assumptions of iso-elastic utility of consumption services, multiplicative habit formation, multiplicative measurement errors, and exponential taste shifters. However as argued in the introduction, there are good empirical reasons for imposing these restrictions when investigating the existence of habit formation in micro consumption data, beyond the fact that they allow us to account for nonparametric measurement errors.

Using the parameter estimates from the model, we develop bounds on the IES and RRA. We find that these parameters display significant variation across individuals and over time. These results are robust across different model specifications. However, there are extensions to the model presented in this paper that can be pursued in future work. One such possibility is to extend the model to allow for more flexible patterns of habit formation. In this paper, we extended the baseline *internal* habit formation model to allow for *external* habit. Another possibility is to allow for more general specifications of internal habits. The current model assumes that internal habit is a function of only the previous period's consumption. The model and estimation method can be extended to include additional lags, but at the cost of a smaller number of time periods from which to recover parameters that dictate consumption patterns.

Because the PSID only collects data on food consumption, its use necessitates the assumption of separability in preferences between food and other nondurables. While Meghir and Weber (1996) suggest that this separability assumption can be the driver behind finding habit formation in food consumption, Carrasco et al. (2005) argue that this result is driven by the presence of time invariant unobserved heterogeneity. In this paper, we control for time invariant unobserved heterogeneity. The use of the PSID also allows for more consistent comparisons of the results in this paper to other important papers in this literature such as Dynan (2000) and Alan and Browning (2010). Alternative data sets, such as the CEX, do not contain a long enough panel to perform a reliable analysis of our model. In addition to separability in preferences between food and other nondurables, our results assume separability in preferences between consumption and leisure. A proper treatment of the effect of labor supply would require modeling and jointly estimating labor supply decisions. This is beyond the scope of the current paper, but remains a part of our research agenda.

That being said, this paper proposes a direct GMM estimator of Euler equations with nonseparabilities in consumption, which can also be used to investigate individual heterogeneity in the IES and RRA. We find that the IES is decreasing and convex in age, and that the RRA is increasing and concave in age, and increasing in education. These findings are consistent with those of other recent empirical, survey-based, and experimental studies. The new findings are that the IES is U shaped in income and the RRA is dome shaped in income. These findings warrant further analysis because of, among other things, their implications for heterogenous consumption and savings responses to various economic policy interventions.

9 Appendix

A Derivation of the moment condition

In order to obtain an expression in terms of observed consumption, we consider equation (3.1) piece by piece and express observed consumption in terms of true consumption and measurement error, as stated above. We start with the first term.

$$
\begin{aligned}
&E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g^o_{it+1}}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}|z_{it}\right]=\\
&E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}}\left(\frac{g_{it+1}}{g^{\alpha}_{it}}\right)^{1-\gamma}\frac{1}{v_{it+1}}\left(\frac{v_{it+1}}{v^{\alpha}_{it}}\right)^{1-\gamma}|z_{it}\right]=\\
&E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}}\left(\frac{g_{it+1}}{g^{\alpha}_{it}}\right)^{1-\gamma}|z_{it}\right]E\left[\frac{1}{v_{it+1}}\left(\frac{v_{it+1}}{v^{\alpha}_{it}}\right)^{1-\gamma}|z_{it}\right]=\\
&E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}}\left(\frac{g_{it+1}}{g^{\alpha}_{it}}\right)^{1-\gamma}|z_{it}\right]\mathcal{A}_1.
\end{aligned}
$$

Under Assumptions 3.3 and 3.4, it can be shown that $\mathcal{A}_1=\exp\{\varsigma^2+\sigma^2\left(\alpha^2(1-\gamma)^2+\gamma^2-\alpha\gamma(1-\gamma)\right)\}$. Hence,

$$
E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g_{it+1}}\left(\frac{g_{it+1}}{g^{\alpha}_{it}}\right)^{1-\gamma}|z_{it}\right]=E\left[\beta\mathcal{A}_1^{-1}(1+r_{it+1})\frac{\varphi_{it+1}}{g^o_{it+1}}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}|z_{it}\right].
$$

The second and the third terms are transformed in the same way to get

$$
\begin{aligned}
E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}\varphi_{it+2}}{g_{it+1}}\left(\frac{g_{it+1}g_{it+2}}{(g_{it}g_{it+1})^{\alpha}}\right)^{1-\gamma}|z_{it}\right] &= E\left[\alpha\beta^2\mathcal{A}_2^{-1}(1+r_{it+1})\frac{\varphi_{it+1}\varphi_{it+2}}{g^o_{it+1}}\left(\frac{g^o_{it+1}g^o_{it+2}}{(g^o_{it}g^o_{it+1})^{\alpha}}\right)^{1-\gamma}|z_{it}\right]\\
E\left[\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g^{\alpha}_{it}}\right)^{1-\gamma}|z_{it}\right] &= E\left[\alpha\beta\mathcal{A}_3^{-1}\varphi_{it+1}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}|z_{it}\right].
\end{aligned}
$$

Again, under Assumptions 3.3 and 3.4 we find that

$$
\begin{aligned}
\mathcal{A}_2 &= \exp\{\varsigma^2+\sigma^2\left(\alpha^2(1-\gamma)^2+\gamma^2+(1-\gamma)(1+\alpha)\right)\},\\
\mathcal{A}_3 &= \exp\{\varsigma^2+\sigma^2\left((1+\alpha+\alpha^2)(1-\gamma)^2\right)\}.
\end{aligned}
$$

The moment condition (3.1) for (unobserved) true consumption is therefore transformed into a moment condition for observed consumption:

$$E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g^o_{it+1}}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\left(\mathcal{A}_1^{-1}-\alpha\beta\mathcal{A}_2^{-1}\varphi_{it+2}\left(\frac{g^o_{it+2}}{g^{o\alpha}_{it+1}}\right)^{1-\gamma}\right)-\right.$$
$$\left.\left(1-\alpha\beta\mathcal{A}_3^{-1}\varphi_{it+1}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\right)|z^o_{it}\right]=0 \qquad \text{(A.1)}$$

where z^o_{it} is a q-dimensional observable subset of z_{it}.

B Proof of Theorem 3.6

Recall that

$$\rho(x^o_{it+2},\theta)=\beta(1+r_{it+1})\frac{\varphi_{it+1}}{g^o_{it+1}}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\left(\kappa_1-\alpha\beta\varphi_{it+2}\left(\frac{g^o_{it+2}}{g^{o\alpha}_{it+1}}\right)^{1-\gamma}\right)$$
$$-\left(\kappa_2-\alpha\beta\kappa_3\varphi_{it+1}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\right).$$

Let $\tilde{\theta}$ be an alternative vector of parameters that satisfy equation (3.2), and define $\Gamma(x^o_{it+2})=\rho(x^o_{it+2},\theta_0)-\rho(x^o_{it+2},\tilde{\theta})$ so that

$$E\left[\Gamma(x^o_{it+2})|z^o_{it}\right]=0.$$

Then by Assumption 3.5.1, for at least one t we have that

$$\Gamma(x^o_{it+2})=0. \qquad \text{(B.1)}$$

Setting $\delta = 0$, $\gamma = 1$, and $\alpha\beta = \kappa_1 = \kappa_2/\kappa_3$, equation (B.1) is solved trivially. Assumption 3.5.3 eliminates this trivial solution. Differentiating equation (B.1) by $(1 + r_{it+1})$ obtains

$$\begin{aligned}
&\beta\frac{\varphi_{it+1}}{g^o_{it+1}}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\left(\kappa_1 - \alpha\beta\varphi_{it+2}\left(\frac{g^o_{it+2}}{g^{o\alpha}_{it+1}}\right)^{1-\gamma}\right)\\
=&\tilde{\beta}\frac{\tilde{\varphi}_{it+1}}{g^o_{it+1}}\left(\frac{g^o_{it+1}}{g^{o\tilde{\alpha}}_{it}}\right)^{1-\tilde{\gamma}}\left(\tilde{\kappa}_1 - \tilde{\alpha}\tilde{\beta}\tilde{\varphi}_{it+2}\left(\frac{g^o_{it+2}}{g^{o\tilde{\alpha}}_{it+1}}\right)^{1-\tilde{\gamma}}\right).
\end{aligned} \tag{B.2}$$

Differentiating equation (B.2) with respect to g^o_{it}, and noting that $g_{it} > 0$ obtains

$$\begin{aligned}
&\alpha(1-\gamma)\beta\frac{\varphi_{it+1}}{g^o_{it}g^o_{it+1}}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\left(\kappa_1 - \alpha\beta\varphi_{it+2}\left(\frac{g^o_{it+2}}{g^{o\alpha}_{it+1}}\right)^{1-\gamma}\right)\\
=&\tilde{\alpha}(1-\tilde{\gamma})\tilde{\beta}\frac{\tilde{\varphi}_{it+1}}{g^o_{it}g^o_{it+1}}\left(\frac{g^o_{it+1}}{g^{o\tilde{\alpha}}_{it}}\right)^{1-\tilde{\gamma}}\left(\tilde{\kappa}_1 - \tilde{\alpha}\tilde{\beta}\tilde{\varphi}_{it+2}\left(\frac{g^o_{it+2}}{g^{o\tilde{\alpha}}_{it+1}}\right)^{1-\tilde{\gamma}}\right).
\end{aligned} \tag{B.3}$$

From equations (B.2) and (B.3), and noting that $g^o_{it} > 0$, we conclude that

$$\alpha(1-\gamma) = \tilde{\alpha}(1-\tilde{\gamma}). \tag{B.4}$$

If $\alpha = \tilde{\alpha} = 0$, then equations (B.2), (B.3), and (B.4) leads to

$$\beta\kappa_1\varphi_{it+1}g^{o-\gamma}_{it+1} = \tilde{\beta}\tilde{\kappa}_1\tilde{\varphi}_{it+1}g^{o-\tilde{\gamma}}_{it+1}. \tag{B.5}$$

Differentiating equation (B.5) by g^o_{it+1} and noting that $g^o_{it+1} > 0$ obtains

$$\gamma\beta\kappa_1\varphi_{it+1}g^{o-\gamma}_{it+1} = \tilde{\gamma}\tilde{\beta}\tilde{\kappa}_1\tilde{\varphi}_{it+1}g^{o-\tilde{\gamma}}_{it+1}. \tag{B.6}$$

Equations (B.5) and (B.6) leads to $\gamma = \tilde{\gamma}$. This and equation (B.4) obtains $\alpha = \tilde{\alpha}$. Thus from equation (B.6), we get $\beta\kappa_1\varphi_{it+1} = \tilde{\beta}\tilde{\kappa}_1\tilde{\varphi}_{it+1}$, from which it is straightforward to show that $\beta\kappa_1 = \tilde{\beta}\tilde{\kappa}_1$ and $\delta = \tilde{\delta}$. This and equation (B.1) obtains $\kappa_2 = \tilde{\kappa}_2$, and $\beta\kappa_3 = \tilde{\beta}\tilde{\kappa}_3$.

Now, if $\alpha > 0$ and $\tilde{\alpha} > 0$, differentiating equation (B.3) with respect to g^o_{it+2} and using equation (B.4) obtains

$$\beta^2\varphi_{it+1}\varphi_{it+2}g^{o-\gamma}_{it+1}g^{o-\gamma}_{it+2} = \tilde{\beta}^2\tilde{\varphi}_{it+1}\tilde{\varphi}_{it+2}g^{o-\tilde{\gamma}}_{it+1}g^{o-\tilde{\gamma}}_{it+2}. \tag{B.7}$$

Differentiating equation (B.7) with respect to g^o_{it+1} and noting $g^o_{it+1} > 0$ obtains

$$-\gamma\beta^2\varphi_{it+1}\varphi_{it+2}g^{o-\gamma}_{it+1}g^{o-\gamma}_{it+2} = -\tilde{\gamma}\tilde{\beta}^2\tilde{\varphi}_{it+1}\tilde{\varphi}_{it+2}g^{o-\tilde{\gamma}}_{it+1}g^{o-\tilde{\gamma}}_{it+2}. \tag{B.8}$$

Equations (B.7) and (B.8) leads to $\gamma = \tilde{\gamma}$. This result and equation (B.4) obtains $\alpha = \tilde{\alpha}$. Thus, from equation (B.7) we get $\beta^2\varphi_{it+1}\varphi_{it+2} = \tilde{\beta}^2\tilde{\varphi}_{it+1}\tilde{\varphi}_{it+2}$, from which it is straightforward to show $\beta = \tilde{\beta}$ and $\delta = \tilde{\delta}$. These and equations (B.2) obtains $\kappa_1 = \tilde{\kappa}_1$. Finally, from equation (B.1), $\kappa_2 = \tilde{\kappa}_2$, and $\kappa_3 = \tilde{\kappa}_3$. Therefore, to summarize, γ, α, δ $\kappa_1\beta$, κ_2, and $\beta\kappa_3$ are identified whether or not $\alpha = 0$. Furthermore, if $\alpha > 0$ then β and $\kappa = (\kappa_1, \kappa_2, \kappa_3)$ are identified.

C Intertemporal Elasticity of Substitution

In this section, we calculate individual-specific intertemporal elasticities of substitution. The individual-specific intertemporal elasticity of substitution can be found from:

$$\frac{1}{IES_{it}} = \left| \frac{\partial \ln \frac{MU_{it}}{MU_{it+1}}}{\partial \ln \frac{c_{it+1}}{c_{it}}} \right| \tag{C.1}$$

where

$$
\begin{aligned}
\frac{MU_{it}}{MU_{it+1}} &= \frac{\frac{\phi_{it}}{c_{it}}\left(\frac{c_{it}}{c_{it-1}^{\alpha}}\right)^{1-\gamma} - \alpha\beta\frac{\phi_{it+1}}{c_{it}}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}}{\frac{\phi_{it+1}}{c_{it+1}}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma} - \alpha\beta\frac{\phi_{it+2}}{c_{it+1}}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha}}\right)^{1-\gamma}} \\
&= \frac{\left(1-\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\right)}{\frac{\varphi_{it+1}}{g_{it+1}}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\left(1-\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}\right)}
\end{aligned}
\tag{C.2}
$$

Taking logs of (C.2) and partial derivatives with respect to $\ln g_{it+1} = \ln \frac{c_{it+1}}{c_{it}}$ we obtain:

$$
\frac{1}{IES_{it}} = \gamma - \frac{\alpha\beta(1-\gamma)\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}}{1-\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}} - \frac{\alpha^2\beta(1-\gamma)\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}}{1-\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}} \tag{C.3}
$$

Because $\alpha \geq 0$ and $\gamma \geq 1$, we obtain the following bounds

$$
\frac{1}{IES_{it}} \geq \gamma, \qquad IES_{it} \leq \frac{1}{\gamma}. \tag{C.4}
$$

These inequalities are strict for $\alpha > 0$ and $\gamma > 1$. In order to derive bounds for the (inverse) IES, we must take into account the measurement errors in observed consumption. To do so, we first rewrite equation (C.3) as follows

$$
\begin{aligned}
\frac{1}{IES_{it}} = \; & \gamma - \alpha\beta(1-\gamma)\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\sum_{j=0}^{\infty}\left(\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j} \\
& -\alpha^2\beta(1-\gamma)\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}\sum_{j=0}^{\infty}\left(\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}\right)^{j} \\
= \; & \gamma - (1-\gamma)\sum_{j=1}^{\infty}\left(\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j} - \alpha(1-\gamma)\sum_{j=1}^{\infty}\left(\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}\right)^{j},
\end{aligned}
\tag{C.5}
$$

which is a valid representation because the assumption of positive marginal utility implies that each term in the infinite sum is between 0 and 1. For the same reason, the dominated convergence theorem applies and we find that

$$E\left[\frac{1}{IES_{it}}|z_{it}\right] = \gamma-(1-\gamma)\sum_{j=1}^{\infty}E\left[\left(\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right] -\alpha(1-\gamma)\sum_{j=1}^{\infty}E\left[\left(\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}}{g_{it+1}^{\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right]. \tag{C.6}$$

Next, for each j we have that

$$\begin{aligned} E\left[\left(\alpha\beta\varphi_{it+1}\left(\frac{g^{o}_{it+1}}{g_{it}^{o\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right] &= E\left[\left(\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j}\left(\left(\frac{v_{it+1}}{v_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right] \\ &= E\left[\left(\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right]E\left[\left(\left(\frac{v_{it+1}}{v_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right] \end{aligned} \tag{C.7}$$

Because $j \geq 1$, Jensen's inequality implies that

$$E\left[\left(\left(\frac{v_{it+1}}{v_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right] \geq \left(E\left[\left(\frac{v_{it+1}}{v_{it}^{\alpha}}\right)^{1-\gamma}|z_{it}\right]\right)^{j} = \mathcal{A}_3^{j}. \tag{C.8}$$

Notice that $\mathcal{A}_3$ is exactly the quantity defined in the derivation of the moment condition in Appendix A. Therefore,

$$E\left[\left(\alpha\beta\varphi_{it+1}\left(\frac{g^{o}_{it+1}}{g_{it}^{o\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right] \geq E\left[\left(\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right]\mathcal{A}_3^{j}. \tag{C.9}$$

Combining equations (C.4), (C.6) and (C.9), we find that

$$\gamma \leq E\left[\frac{1}{IES_{it}}|z_{it}\right] \leq \gamma-(1-\gamma)\sum_{j=1}^{\infty}E\left[\left(\mathcal{A}_3^{-1}\alpha\beta\varphi_{it+1}\left(\frac{g^{o}_{it+1}}{g_{it}^{o\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right] -\alpha(1-\gamma)\sum_{j=1}^{\infty}E\left[\left(\mathcal{A}_3^{-1}\alpha\beta\varphi_{it+2}\left(\frac{g^{o}_{it+2}}{g_{it+1}^{o\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right], \tag{C.10}$$

with strict inequalities if $\alpha > 0$ and $\gamma > 1$. Again, by Jensen's inequality we have that $(E[1/IES_{it}|z_{it}])^{-1} \leq E[IES_{it}|z_{it}]$. The corresponding bound for $E[IES_{it}|z_{it}]$ is given by

$$\begin{aligned} \frac{1}{\gamma} \geq E\left[IES_{it}|z_{it}\right] \geq \quad & \left(\gamma - (1-\gamma)\sum_{j=1}^{\infty} E\left[\left(\mathcal{A}_3^{-1}\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}^{o}}{g_{it}^{o\alpha}}\right)^{1-\gamma}\right)^j |z_{it}\right]\right. \\ & \left. -\alpha(1-\gamma)\sum_{j=1}^{\infty} E\left[\left(\mathcal{A}_3^{-1}\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}^{o}}{g_{it+1}^{o\alpha}}\right)^{1-\gamma}\right)^j |z_{it}\right]\right)^{-1}. \end{aligned} \tag{C.11}$$

Under the assumption that measurement errors are distributed log-normally as in Section 3, the inverse IES is point identified. To see this, note that under this assumption

$$E\left[\left(\left(\frac{v_{it+1}}{v_{it}^{\alpha}}\right)^{1-\gamma}\right)^j |z_{it}\right] = \mathcal{A}_3^{j^2}. \tag{C.12}$$

Then, straightforward calculations give

$$\begin{aligned} E\left[\frac{1}{IES_{it}}|z_{it}\right] = \quad & \gamma - (1-\gamma)\sum_{j=1}^{\infty} E\left[\mathcal{A}_3^{-j^2}\left(\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}^{o}}{g_{it}^{o\alpha}}\right)^{1-\gamma}\right)^j |z_{it}\right] \\ & -\alpha(1-\gamma)\sum_{j=1}^{\infty} E\left[\mathcal{A}_3^{-j^2}\left(\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}^{o}}{g_{it+1}^{o\alpha}}\right)^{1-\gamma}\right)^j |z_{it}\right]. \end{aligned} \tag{C.13}$$

The parametric distributional assumption for measurement errors does not entail point identification of the IES, but does reduce the bound as follows

$$\begin{aligned} \frac{1}{\gamma} \geq E\left[IES_{it}|z_{it}\right] \geq \quad & \left(\gamma - (1-\gamma)\sum_{j=1}^{\infty} E\left[\mathcal{A}_3^{-j^2}\left(\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}^{o}}{g_{it}^{o\alpha}}\right)^{1-\gamma}\right)^j |z_{it}\right]\right. \\ & \left. -\alpha(1-\gamma)\sum_{j=1}^{\infty} E\left[\mathcal{A}_3^{-j^2}\left(\alpha\beta\varphi_{it+2}\left(\frac{g_{it+2}^{o}}{g_{it+1}^{o\alpha}}\right)^{1-\gamma}\right)^j |z_{it}\right]\right)^{-1}. \end{aligned} \tag{C.14}$$

This bound is typically more narrow than in equation (C.14) because $\mathcal{A}_3 > 1$ so that $\mathcal{A}_3^{-j^2} < \mathcal{A}_3^{-j}$.

D Relative Risk Aversion

In this section, we calculate individual-specific relative risk aversion parameters. These coefficients correspond to curvature and are closely related to the elasticities of the marginal utility of consumption with respect to consumption. Individual-specific relative risk aversion is defined as:

$$RRA_{it} = -c_{it}\frac{\Lambda_{it}^{cc}}{\Lambda_{it}^{c}} \tag{D.1}$$

where $\Lambda_{it}^{c} = MU_{it}$ and $\Lambda_{it}^{cc} = \frac{\partial MU_{it}}{\partial c_{it}}$. Consequently, the risk aversion parameters implied by our model are given by:

$$RRA_{it} = \frac{\gamma - \alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma} - \alpha^2\beta(1-\gamma)\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}}{1-\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}} \tag{D.2}$$

If the observed consumption c^o is contaminated with measurement errors, we must take these into account when calculating individual-specific RRAs. We first rewrite equation () as follows

$$RRA_{it} = \gamma - (1+\alpha)(1-\gamma)\sum_{j=1}^{\infty}\left(\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}}{g_{it}^{\alpha}}\right)^{1-\gamma}\right)^{j}. \tag{D.3}$$

The same arguments as those used in the previous section also validate this expression. Therefore, calculations similar to those in the previous section lead us to the inequality

$$\gamma \leq E\left[RRA_{it}|z_{it}\right] \leq \gamma - (1+\alpha)(1-\gamma)\sum_{j=1}^{\infty}E\left[\left(\mathcal{A}_3^{-1}\alpha\beta\varphi_{it+1}\left(\frac{g_{it+1}^{o}}{g_{it}^{o\alpha}}\right)^{1-\gamma}\right)^{j}|z_{it}\right], \tag{D.4}$$

with strict inequality if $\alpha > 0$ and $\gamma > 1$. Furthermore, if the log-normal assumption on measurement errors is maintained, then we find that

$$E\left[RRA_{it}|z_{it}\right] = \gamma - (1+\alpha)(1-\gamma)\sum_{j=1}^{\infty} E\left[\mathcal{A}_3^{-j^2}\left(\alpha\beta\varphi_{it+1}\left(\frac{g^o_{it+1}}{g^{o\alpha}_{it}}\right)^{1-\gamma}\right)^j |z_{it}\right]. \tag{D.5}$$

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